

MAGNETIC FIELDS AND THE EW PHASE TRANSITION*

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We review the motivation for, lattice results on, and some implications of, external magnetic fields present at the time of the cosmological electroweak phase transition.

1 Introduction

The existence of galactic magnetic fields today may imply the existence of primordial seed fields in the Early Universe. In order to get large enough length scales, it seems conceivable^{2,3} that the seed fields should have a correlation length at least of the order of the horizon radius at the electroweak (EW) epoch, $T \sim 100$ GeV. Such large length scales could possibly be produced during the inflationary period of Universe expansion.⁴

Whether or not seed fields at the horizon scale are present at $T \sim 100$ GeV, it is in any case clear that the existence of magnetic fields at somewhat smaller length scales cannot be excluded. Indeed, magnetohydrodynamics,

$$\frac{\partial \mathbf{B}_Y}{\partial t} = \frac{1}{\sigma} \nabla^2 \mathbf{B}_Y + \nabla \times (\mathbf{v} \times \mathbf{B}_Y), \quad (1)$$

tells that magnetic fields diffuse away at scales $l \lesssim (t/\sigma)^{1/2} \sim (M_{\text{Pl}}/T)^{1/2} T^{-1}$. At the EW epoch this gives $l_{\text{EW}} \sim 10^7/T$. Since diffusion continues after the electroweak epoch, constraints from primordial nucleosynthesis and CMBR only concern fields at scales larger than these, by factors $(T_{\text{EW}}/T_{\text{nucl.synth.}})^{1/2} \sim 10^3$ and $(T_{\text{EW}}/T_{\text{recomb.}})^{1/2} \sim 10^5$, respectively.

A further question is the magnitude of magnetic fields. An equipartition argument would say that only a small fraction of the total (free) energy density can be in magnetic fields. This leads to $B_Y/T^2 \lesssim 2 \dots 3$.

In conclusion, there could well be essentially homogeneous and macroscopic ($l_{\text{EW}} \gg T^{-1}$) magnetic fields around at $T \sim 100$ GeV, with a magnitude $B_Y/T^2 \sim 1$. The purpose of this talk is to review how such fields would affect the cosmological EW phase transition.

2 The perturbative EW phase diagram in an external field

To be specific, we study here a system with a fixed hypercharge flux $\Phi_{B_Y} = \int d\mathbf{s} \cdot \mathbf{B}_Y$. The properties of a system with a fixed field strength can be obtained

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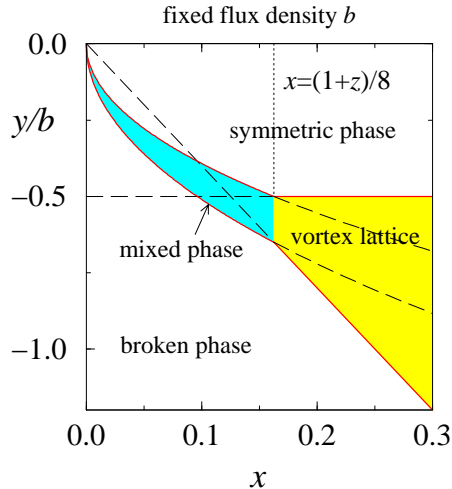


Figure 1: The tree-level phase diagram of the electroweak theory in a magnetic flux.

with a Legendre transformation. The thermodynamics depends essentially on three dimensionless parameters,

$$x \sim \frac{1}{8} \frac{m_H^2}{m_W^2}, \quad y \sim 4.5 \frac{T - T_0}{T_0}, \quad b \sim 2.0 \frac{\Phi_{B_Y}}{(\text{area})} \frac{1}{T^2}, \quad (2)$$

where T_0 equals the critical temperature up to radiative corrections and the numerical factors are there for technical reasons. Thus, one can tune x (Higgs mass), y (temperature) and b (magnetic flux density). A further parameter, $z \sim \tan^2 \theta_W$, is fixed to a constant value $z \approx 0.3$.

In the space of these variables, the tree-level phase diagram of the theory is as shown in Fig. 1. At small x , there is a first order phase transition,⁵ the stronger the larger b . (1-loop corrections⁶ do not change this statement in any essential way.) In the present ensemble the transition takes place through a mixed phase, containing simultaneously macroscopic regions of the symmetric and broken phases. At large x , the situation is different: the ground state solution of the classical equations of motion is a vortex lattice, or the “Ambjørn-Olesen phase”.⁷

3 The non-perturbative phase diagram

The perturbative phase diagram discussed above need not be reliable at large Higgs masses, $x \gtrsim 0.1$, since the dynamics of the theory is completely non-

perturbative there.⁸ Thus, the existence of a vortex lattice is also not obvious.

It is perhaps illuminating to consider more generally whether a vortex lattice phase can really exist in three dimensions in a fluctuating system. We will skip the question of whether such a phase can exist in a strict mathematical sense, and consider the issue from a more practical point of view. Then, it is clear that a lattice phase could exist in principle, since an Abrikosov vortex lattice and a related 1st order melting transition have been observed even experimentally in a very similar system, a superconductor. There is a phenomenological test, called the Lindemann criterion, for when a lattice phase can be observed: the requirement is that the root mean square fluctuation of the flux line position around its average, be sufficiently smaller than the lattice spacing.⁹ In superconductors this criterion can be satisfied, since the fluctuations can be reduced by going to lower and lower temperatures. In the present system the requirement is more difficult to satisfy, since, for fixed b , one cannot go to arbitrarily low temperatures within the vortex phase, see Fig. 1. However, a quantitative estimate of the average fluctuations is lacking at the moment, and we thus have to turn to lattice experiments.

A fixed magnetic flux can easily be implemented on the lattice, by using a 3d effective field theory^{10,11,12} and by modifying the boundary conditions related to the hypercharge $U_Y(1)$ field:

$$\Phi_{B_Y} = \int d\mathbf{s} \cdot \mathbf{B}_Y = \int dx_1 dx_2 F_{12} = \oint ds_i A_i. \quad (3)$$

Otherwise the simulations proceed very much like without a magnetic field.¹¹

There is one important new lattice artifact to be mentioned. Indeed, with the ensemble used, there can be a mixed phase in the system, as shown in Fig. 1. A mixed phase implies the existence of surfaces, which cost energy. Only at large enough volumes does the bulk free energy win over the surface energy. Thus, for finite lattice sizes, small values of x , corresponding to a strong 1st order transition and large surface tension, do not display a mixed phase.[†] For large x , $x \geq (1+z)/8$, on the other hand, the surface energy becomes negative as in type II superconductors and the strong volume dependence disappears. Hence, these volume artifacts do not appear in the vortex lattice phase.

The basic results of the lattice simulations are as follows:

First order regime, $x < (1+z)/8$: the transition is qualitatively similar to that in perturbation theory, Fig. 1: a strong 1st order transition. However, for a given lattice volume, one has to go to a large Higgs mass where the surface tension is sufficiently small to see the mixed phase. A phase diagram determined from finite volume lattices is shown in Fig. 2.

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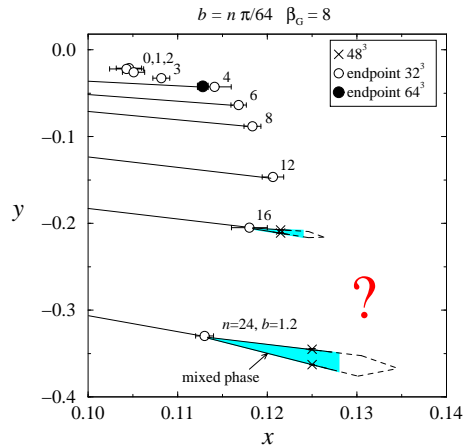


Figure 2: The non-perturbative phase diagram of the electroweak theory in a magnetic flux, for finite volumes. The behaviour in the region of the question mark is still unknown.

Intermediate regime, $x \sim (1+z)/8$: when x is increased, the phase transition gets weaker and finally ends, and the endpoint looks qualitatively similar to the one described in Ref.⁸, in the absence of a magnetic field.

Vortex lattice regime, $x > (1+z)/8$: for the magnetic fields studied so far, we have not observed the vortex lattice phase! The individual or averaged configurations do not display any non-trivial structure, and there are no qualitative changes in any of the observables measured.

4 Conclusions

We have seen that an external magnetic field makes a 1st order transition stronger, but there is still no transition in the Standard Model for $x \gtrsim 0.14$ (at least for $B_Y/T^2 \lesssim 0.5$), corresponding to $m_H \gtrsim 90$ GeV. Larger fields have not yet been conclusively studied. We have not observed a vortex lattice phase with qualitatively new properties at these magnetic fields, even though the solution of the classical equations of motion displays such a phase.

In the MSSM, there can be a 1st order transition for the experimentally allowed Higgs masses. Then an external magnetic field might have implications, e.g., for baryogenesis: the bubble dynamics and the real time history of the transition get modified, and the external magnetic field can couple also directly to fermion number non-conservation.⁵

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